

International GCSE Further Pure Mathematics – Paper 1 mark scheme

| Paper 1 | | Working | Answer | Mark | AO | Notes |
|---------|-----|---|--------|----------|----|--|
| 1 | (a) | <p>Graph (a) shows two lines on a Cartesian coordinate system. The first line is labeled $2x + 3y = 8$ and has a y-intercept at $y = \frac{8}{3}$. The second line is labeled $2y = 4x + 1$ and has a y-intercept at $y = \frac{1}{2}$. The two lines intersect at a point. Dashed lines from this intersection point indicate its coordinates are $x = \frac{8}{3}$ and $y = \frac{1}{2}$. A region labeled R is shaded between the two lines and the y-axis.</p> | | B1 B1 | 1 | One correct line Both correct lines |
| | (b) | <p>Graph (b) shows two vertical lines on a Cartesian coordinate system. The first line is labeled $x = -\frac{1}{4}$ and the second line is labeled $x = 4$. A region labeled R is shaded between these two lines.</p> | | B1 B1 | 1 | Correct line $x = 2$ drawn Correct region shaded in or out. |
| | | | | (4) | | |

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|----------|--|-------------------------|---------------------------------|--------|--|
| 2 (a) | $\pi = 6\theta \Rightarrow \theta = \frac{\pi}{6} \Rightarrow AOB = \frac{5\pi}{6} = (150^\circ)$ $AB = \sqrt{10^2 + 6^2 - 2 \times 10 \times 6 \times \cos\left(\frac{5\pi}{6}\right)} = 15.4894\dots = 15.5 \text{ cm}$ | 15.5 (cm) | B1 M1A1 | 1 2 | Accept working in degrees $AOB = 150^\circ$ |
| (b) | $\text{Area} = \frac{1}{2} \times 10 \times 6 \times \sin \frac{5\pi}{6} + \frac{\pi}{6} \times \frac{6^2}{2} = 24.424\dots$ | 24.4 (cm ²) | M1M1A1 | 2 | |
| | <p>ALTERNATIVE</p> $\text{Area} = \frac{1}{2} \times 10 \times 6 \times \sin \frac{5\pi}{6} + \frac{1}{2} \times \pi \times 6 = 24.424\dots$ | | M1M1A1 (6) | | |
| 3 | $2 \cos(2\theta + 30) + \frac{\sin(2\theta + 30)}{\cos(2\theta + 30)} = 0 \Rightarrow 2 \cos^2(2\theta + 30) + \sin(2\theta + 30) = 0$ $\Rightarrow 2 - 2 \sin^2(2\theta + 30) + \sin(2\theta + 30) = 0$ $\sin(2\theta + 30) = \frac{1 \pm \sqrt{1 - 4 \times 2 \times (-2)}}{2 \times 2} = 1.2807\dots, -0.7807\dots$ $2\theta + 30 = -51.33167\dots, 231.33167, 308.66833$ $\theta = 100.7, 139.3$ | $\theta = 100.7, 139.3$ | M1 M1A1 M1 A1A1 (6) | 2 3 | Solves 3 TQ Finds one angle from their 3TQ |

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| 4 | | | | | |
| (a) | $v = 0$ so $4t^2 - 19t + 12 = 0 \Rightarrow (4t - 3)(t - 4) = 0 \Rightarrow t = \frac{3}{4}, 4$ | $t = \frac{3}{4}, 4$ | M1A1 | 1 | |
| (b) | $s = \int 4t^2 - 19t + 2 dt = \frac{4t^3}{3} - \frac{19t^2}{2} + 12t + c$ when $t = 0, s = -4 \Rightarrow c = -4$ When $t = 6, s = \frac{4 \times 6^3}{3} - \frac{19 \times 6^2}{2} + 12 \times 6 - 4 = 14$ | 14 | M1M1A1 | 2 | |
| (c) | $a = \frac{dv}{dt} = 8t - 19 \Rightarrow 8t - 19 = 0 \Rightarrow t = \frac{19}{8}$ | $t = \frac{19}{8}$ | A1 M1M1A1 (9) | 3 | |
| 5 | | | | | |
| (a) | $2x + y = 13 \Rightarrow y = 13 - 2x$ $S = 4x^2 + (13 - 2x)^2 = 4x^2 + 169 - 52x + 4x^2 = 8x^2 - 52x + 169$ | $8x^2 - 52x + 169$ | M1A1 | 1 | |
| (b) | $\frac{dS}{dx} = 16x - 52 \quad \frac{dS}{dx} = 0 \Rightarrow 16x - 52 = 0 \Rightarrow x = \frac{13}{4}$ $\frac{d^2S}{dx^2} = 16 \quad 16 > 0,$ | $x = \frac{13}{4}$ Hence minimum | M1M1A1 B1 | 2,3 | |
| (c) | $S = 8 \times \left(\frac{13}{4}\right)^2 - 52 \times \frac{13}{4} + 169 = \frac{169}{2} = 84.5$ | $\frac{169}{2} = 84.5$ | M1A1 (9) | 3 | |

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|------------|--|-------------|---------------------------------|-------------|-------|-------|---|---|---|---|-------------|----------|-------------|---|-------|--|--|---|--|
| 6 | $\frac{dy}{dx} = e^x(x^2 - 3x) + e^x(2x - 3) \Rightarrow e^x(2x - 3) = \frac{dy}{dx} - y$ $\frac{d^2y}{dx^2} = e^x(x^2 - 3x) + e^x(2x - 3) + e^x(2x - 3) + 2e^x = y + 2\left(\frac{dy}{dx} - y\right) + 2e^x$ $2e^x = \frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx} - y\right) - y \Rightarrow 2e^x = y - 2\frac{dy}{dx} + \frac{d^2y}{dx^2} *$ | | MIMIAI MIAI MIMIAI (8) | 4 4 | | | | | | | | | | | | | | | |
| 7 | <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>3</td> <td>3.83</td> <td>5</td> <td>6.66</td> <td>9</td> <td>12.31</td> </tr> </tbody> </table> | x | 0 | 1 | 2 | 3 | 4 | 5 | y | 3 | 3.83 | 5 | 6.66 | 9 | 12.31 | | | 1 | |
| x | 0 | 1 | 2 | 3 | 4 | 5 | | | | | | | | | | | | | |
| y | 3 | 3.83 | 5 | 6.66 | 9 | 12.31 | | | | | | | | | | | | | |
| (a) | | | | | | | | | | | | | | | | | | | |
| (b) | All points plotted within an accuracy of half of a square. A smooth curve drawn through their points | | | 1 | | | | | | | | | | | | | | | |
| (c) | $\log_2(4x - 6)^2 - x = 2 \Rightarrow 2 \log_2(4x - 6) = x + 2 \Rightarrow \log_2(4x - 6) = \frac{x}{2} + 1$ $\Rightarrow 4x - 6 = 2^{\left(\frac{x}{2} + 1\right)} \Rightarrow 4x - 5 = 2^{\left(\frac{x}{2} + 1\right)} + 1$ <p>Line $y = 4x - 5$ drawn on graph \Rightarrow so $x = 2.8(36)$</p> | | | 2 | | | | | | | | | | | | | | | |
| | | $x = 2.8$ | MIAI (8) | | | | | | | | | | | | | | | | |

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| 8 | | | | | |
| (a) | $a = S_1 = 2 \times 1 \times (1+3) = 8$ | $a = 8$ | B1 | 1 | |
| (b) | $S_2 = 2 \times 2 \times (2+3) = 20$ $S_2 = a + T_2 \Rightarrow T_2 = S_2 - a = 20 - 8 = 12$ $d = 12 - 8 = 4$ | $d = 4$ | M1AI | 1 | |
| (c) | <p>Uses given formula for $S_n = 2n(n+3)$ and formula for nth term $T_n = a + (n-1)d$</p> $6[2(n-4)(n-4+3)] = 7[8+(n+3-1)4]$ $\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ $(n-8)(3n+2) = 0$ $\Rightarrow n = 8, \left(n = -\frac{2}{3} \right)$ <p>ALT</p> <p>Using formula $S_n = \frac{n}{2}(2a + (n-1)d)$</p> $6 \left[\frac{(n-4)}{2} (2 \times 8 + ((n-4)-1)4) \right] = 7[8+(n+3-1)4]$ $\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ $(n-8)(3n+2) = 0$ $\Rightarrow n = 8, \left(n = -\frac{2}{3} \right)$ | $n = 8$ | M1M1 M1AI | 2, 3 | |
| | | | M1AI | | |
| | | | M1M1 M1AI | 2, 3 | |
| | | | M1AI | | |
| | | $n = 8$ | (9) | | |

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| 9 (a) | $\alpha + \beta = -\frac{7}{3} \quad \alpha\beta = -2 = -\frac{6}{3} \text{ so } a = 3, b = 7 \text{ and } c = -6$ <p>Hence quadratic equation $\Rightarrow 3x^2 + 7x - 6 = 0$ oe with integer coefficients</p> | | B1B1 M1A1 | 1 | |
| (b) | $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = (\alpha + \beta)^2 - 4\alpha\beta = \left(-\frac{7}{3}\right)^2 - 4 \times -2 = \frac{121}{9}$ $\alpha > \beta \text{ so } \alpha - \beta = \frac{11}{3} *$ | | M1A1 | 3 | |
| (c) | <p>Sum</p> $\frac{\alpha + \beta}{\alpha} + \frac{\alpha - \beta}{\beta} = \frac{\beta(\alpha + \beta) + \alpha(\alpha - \beta)}{\alpha\beta} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\left(-\frac{7}{3}\right)^2 - 2 \times -2}{-2} = -\frac{85}{18}$ <p>Product</p> $\frac{(\alpha + \beta)}{\alpha} \times \frac{(\alpha - \beta)}{\beta} = \frac{\left(-\frac{7}{3}\right) \times \left(\frac{11}{3}\right)}{-2} = \frac{77}{18}$ <p>Equation $18y^2 + 85y + 77 = 0$ oe with integer coefficients</p> | | M1M1A1 M1A1 M1A1 | 3 | |
| | | | (13) | | |

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|-----------|--|--------------|----------------|----|---|
| 10 | Let M be the midpoint of diagonals AC or DB | | | | |
| (a) | $AC = \sqrt{8^2 + 8^2} = 8\sqrt{2} \Rightarrow AM = 4\sqrt{2}$ $h = \sqrt{12^2 - (4\sqrt{2})^2} = \sqrt{112} = (4\sqrt{7})$ | $4\sqrt{7}$ | MIMIAI | 1 | |
| (b) | $\tan^{-1}\left(\frac{4\sqrt{7}}{4\sqrt{2}}\right) = 61.87449\dots \approx 61.9^\circ$ | 61.9° | MIAI | 1 | Or any equivalent trigonometry |
| (c) | Let N be the midpoint of AB | | | | |
| (d) | <p>Angle the plane AOB makes with horiz = $\tan^{-1}\left(\frac{4\sqrt{7}}{4}\right) = 69.295\dots \approx 69.3^\circ$</p> <p>By using the symmetrical properties of the pyramid</p> <p>Let S be the perpendicular from P to diagonal AC Let R be the perpendicular from S to side BC</p> <p>In triangle $PSR \rightarrow PR = \sqrt{(2\sqrt{17})^2 - 2^2} = 8$ In triangle $PRQ \rightarrow PQ = \sqrt{8^2 + 4^2} = 4\sqrt{5}$</p> | 69.3° | MIAI | 3 | Or any equivalent trigonometry |
| (e) | <p>Length $AQ = \sqrt{8^2 + 6^2} = 10$</p> <p>Angle of $PQA = \cos^{-1}\left(\frac{10^2 + (4\sqrt{5})^2 - 6^2}{2 \times 10 \times 4\sqrt{5}}\right) = 36.39124\dots \approx 36.4^\circ$</p> | 36.4° | MIAIAI (15) | 3 | Or any equivalent system of right angle triangles |

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| (d) | <p>ALTERNATIVE without using the symmetrical properties of the pyramid</p> $\cos OAB = \frac{12^2 + 8^2 - 12^2}{2 \times 8 \times 12} = \frac{1}{3}$ $\Rightarrow PB = \sqrt{6^2 + 8^2 - 2 \times 6 \times 8 \times \frac{1}{3}} = 2\sqrt{17}$ <p>In triangle <i>PBC</i></p> $PC = \sqrt{6^2 + (8\sqrt{2})^2 - 2 \times 6 \times (8\sqrt{2}) \times \cos\left(\tan^{-1}\left(\frac{\sqrt{7}}{\sqrt{2}}\right)\right)} = 10$ $\Rightarrow \text{Angle } PBC = \cos^{-1}\left(\frac{8^2 + 68 - 10^2}{2 \times 8 \times 2\sqrt{17}}\right) = 75.9637\dots^\circ$ <p>In triangle <i>PBQ</i>;</p> $PQ = \sqrt{6^2 + 68 - 2 \times 6 \times 2\sqrt{17} \times \cos 75.96375\dots} = 4\sqrt{5}$ | $4\sqrt{5}$ | M1 M1 M1A1 | 3 | |

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|-----------|--|--------------------------|-------------|------|-------|
| 11 | Mark parts (i) and (ii) together | | | 1, 3 | |
| (a) | $y = \int 6x^2 - 26x + 12 dx = \left[\frac{6x^3}{3} - \frac{26x^2}{2} + 12x + c \right]$ <p>At the point $(-1, 0)$</p> $0 = 2(-1)^3 - 13(-1)^2 + 12(-1) + C \Rightarrow C = 27$ | M1 | | | |
| | $(2x^3 - 13x^2 + 12x + 27) \div (x+1) = 2x^2 - 15x + 27 = (2x-9)(x-3)$ $\Rightarrow a = \frac{9}{2}, b = 3$ | M1M1A1 B1B1 | | | |
| (b) | $\text{Area} = \int_0^3 2x^3 - 13x^2 + 12x + 27 dx + \left \int_3^9 2x^3 - 13x^2 + 12x + 27 dx \right =$ $\left[\frac{2}{4}x^4 - \frac{13}{3}x^3 + \frac{12}{2}x^2 + 27x \right]_0^3 + \left[\frac{2}{4}x^4 - \frac{13}{3}x^3 + \frac{12}{2}x^2 + 27x \right]_3^9 = \frac{2043}{32}$ <p>So Area = $\frac{2043}{32}$</p> | MIM1 | 3 | | |
| | | Area = $\frac{2043}{32}$ | A1M1A1 | | |
| | | Total | (13) | | |
| | | Total | 100 | | |

